Almost Everywhere Local Lipschitz Continuity of the Minimum Time Function and Smooth Lyapunov Functions for a Class of Step Two Carnot Groups

Pierpaolo Soravia*

It is well known that the minimum time function of a controllable nonlinear system that does not satisfy at the origin Petrov's condition is only locally Hoelder continuous in its domain, in general. An interesting question is the structure of the set where local Lipschitz continuity fails. We will discuss a sufficient condition for local Lipschitz continuity of the minimum time function in a subset of its domain when we can find a smooth Lyapunov function as the (super-)solution of the Aronsson equation, a second order quasi linear partial differential equation, known in the calculus of variations literature. We show how this idea applies to a class of step two Carnot groups, a special family of symmetric control systems, and allows to find continuous feedbacks driving the system to the origin in finite time, for any starting point except on a singular manifold. The Lyapunov functions that we define and construct make the Hamitonian monotone along the trajectories of the hamiltonian dynamics. The corresponding minimum time function is then locally Lipschitz continuous away from the singular manifold, despite classical results show that it should only be Hoelder continuous.

*Dipartimento di Matematica, Università di Padova, soravia@math.unipd.it